Cognitive WMNs: A Distributed Mechanism for Leasing Cellular Network Bandwidth

S. Toumpis*, I. Tselekounis*, G. D. Stamoulis*, H. Meyer†, A. Hess†, K. A. Hummel†
* Informatics Department, Athens University of Economics and Business (AUEB), Greece
† Entertainment Computing Group (formerly Distributed Systems Group), Univ. of Vienna, Austria

***Abstract*—We study flow optimization in cognitive Wireless Mesh Networks (WMN) that lease bandwidth from a collocated cellular network (CN). We develop a distributed mechanism that such WMNs can use to optimize their traffic given their own utilities and costs and the costs that the CN is charging for the use of its channels. We evaluate the mechanism in a synthetic scenario using the topology of an actual deployed WMN and time-varying traffic requirements induced from actual vehicle mobility traces. Simulations show that the mechanism can track well the changes in the problem parameters, continuously keeping close to the optimum operation point.**

I. Introduction

Wireless Mesh Networks (WMN) have recently become a very popular type of infrastructure, serving a multitude of applications such the offering of alternative IP interconnection, supporting vehicular networks, etc. However, in WMNs, a number of factors (use of the ISM band, multihop transmissions, bulky data, etc.) contribute to an often severe bandwidth shortage. One way of alleviating the bandwidth shortage is to adopt a cognitive network approach [1] under which the WMN uses bandwidth that is available to nearby cellular networks (CNs), whenever it happens to be underutilized by them. Indeed, the fluctuating pattern of cellular type traffic, where voice predominates, implies that, at least for parts of the day and most of the night, large portions of the bandwidth remain unused [2].

In Section II we define a system model that consists of a WMN and a set of CN channels that are available for lease. The topology and the technology used are such, that for each CN channel there is a subset of WMN links that will interfere with the operation of that channel, if traffic passes through them. Furthermore, each WMN link may interfere with more than one channel. Each WMN node derives a utility value by inserting flow in the network if it is a source (or removing flow from the network, if it is a sink). This utility is a concave function of the traffic inserted (or removed). Each channel is available for use but for a cost which is a convex function of its utilization by the WMN. Finally, the use of each WMN link is also subject to some convex cost. We show that the point of socially optimal operation, that maximizes the sum of the utilities of all nodes, minus the costs of all channels and links, can be found by solving a convex program.

In Section III we show that this problem can be solved through a distributed mechanism executed by agents located throughput the network. The mechanism provably establishes the point of optimal operation when the problem parameters do not change. For the case where the parameters of the problem change, for example due to mesh client mobility and/or the fluctuating availability of the cellular bandwidth, we introduce a properly modified version of the mechanism that produces traffic flows that track closely the time-varying optimal one.

In Section IV we perform a preliminary evaluation of the mechanism in a synthetic scenario using the topology of the Funkfeuer WMN [3] currently deployed in Vienna and when the traffic that the WMN must support originates from vehicular mesh clients whose mobility is based on actual vehicular mobility traces collected from the same city. We show that the distributed mechanism succeeds in keeping the network close to its optimum operation point, despite the fact that this is constantly changing due to the mobility of the clients. In Section V we present an overview of related work in the field of cognitive WMNs. We conclude in Section VI.

II. System Model

We model the WMN in terms of a set $\mathcal{N}$ of N **nodes**, indexed by $n = 1, \ldots, N$ and a set $\mathcal{L}$ of L directed **links**, indexed by $l = 1, \ldots, L$. Let $s(l) \in \mathcal{N}$ be the node from which the link starts, i.e., the transmitter, and $e(l) \in \mathcal{N}$ be the node where the link ends, i.e., the receiver. There may be multiple links starting and ending at the same pair of nodes.

The **flow** of data packets along link l, measured in bps, is denoted by x_l. Let $\mathbf{x} = (x_l : l \in \mathcal{L})$ be the **flow vector**. We require $0 \leq x_l \leq x_l^{\max}$. The value $x_l^{\max}$ is a capacity constraint for that link and depends on the transceiver technology used, and perhaps also arbitrary constraints set by the modeler. Let $\mathbf{x}^{\max} = (x_l^{\max} : l \in \mathcal{L})$. Based on the above, $\mathbf{0}_L \leq \mathbf{x} \leq \mathbf{x}^{\max}$, componentwise, where $\mathbf{0}_L$ is a L-sized vector with zero components. We assume, for each link l, a convex **link cost function** $h_l(x_l)$, which models the cost of having flow x_l through link l. The link cost function can model a variety of costs, for example energy costs.

Let

$$s_n = \sum_{l \in \mathcal{L}: n = s(l)} x_l - \sum_{l \in \mathcal{L}: n = e(l)} x_l$$

be the **divergence** of the flow vector at node n. The divergence expresses the rate, in bps, with which the node inserts data in the network (if positive) or removes data from the network (if negative) so that information is conserved, given the flows of traffic along ingress and egress links. Let $\mathbf{s} = (s_n : n \in \mathcal{N})$

be the **divergence vector**. We assume that the divergence is subject to optimization, but must stay within an upper and a lower bound, i.e., $s_n^{\min} \leq s_n \leq s_n^{\max}$. These bounds may be used to express restrictions on the technology or restrict the behavior of the nodes. Let $\mathbf{s}^{\max} = (s_n^{\max} : n \in \mathcal{N})$, $\mathbf{s}^{\min} = (s_n^{\min} : n \in \mathcal{N})$. Based on the above, $\mathbf{s}^{\min} \leq \mathbf{s} \leq \mathbf{s}^{\max}$. For each node n, there is a concave **utility function** $U_n(s_n)$ that expresses the utility the node derives by having divergence s_n.

We define a **channel** to be the whole bandwidth used by a specific cell of a nearby CN. Therefore, there is one channel for each cell, and so in the following the words channel and cell will often be used interchangeably. Let $\mathcal{C}$ be the set of channels, indexed by $c = 1, \ldots, C$. Let r_c be the **reservation level** of channel c, and $\mathbf{r} = (r_c : c \in \mathcal{C})$ be the **reservation vector**. Reservation levels are unitless, and express which fraction (of time, frequency, etc., depending on the technology used) of the respective channel is occupied by the traffic of the WMN links. Each reservation level r_c must be nonnegative and less than a maximum value $r_c^{\max}$ that specifies the maximum fraction of the channel the CN is prepared to lease. Let $\mathbf{r}^{\max} = (r_c^{\max} : c \in \mathcal{C})$, so that $\mathbf{0}_C \leq \mathbf{r} \leq \mathbf{r}^{\max}$, where $\mathbf{0}_C$ is a C-sized vector with zero components. For each of the C channels, we define a convex **channel cost function** $f_c(r_c)$. The channel cost function combines two terms: the cost incurred to the CN by not having the channel available, which must be offset by the WMN, and also a term corresponding to the profit the CN expects.

When a node transmits over a link, we expect that the operation of possibly more than one channel will be interfered with, and each of these must be reserved. In addition, we expect that each channel can possibly be interfered with by more than one link. This arbitrary coupling between links and channels is modeled as follows: let $R_{cl} \geq 0$, the **reservation coefficient** for channel c and link l. Let the **reservation matrix** $R = \{R_{cl}\}$ of size $C \times L$. We require that

$$\mathbf{r} = R\mathbf{x} \Leftrightarrow r_c = \sum_{l=1}^{L} R_{cl} x_l, \quad \forall c \in \mathcal{C}.$$

The precise value of the coefficients will depend on the topology, the transceiver technology used, and various other aspects of the hardware involved. Establishing the precise value for the coefficients goes beyond the scope of this work, however we stress that our model can capture a large variety of situations.

We can now specify our flow optimization problem:

Problem 1: Utility minus Costs (U-C) Maximization

$$\text{maximize:} \quad \sum_{n=1}^{N} U_n(s_n) - \sum_{c=1}^{C} f_c(r_c) - \sum_{l=1}^{L} h_l(x_l), \tag{1}$$

$$\text{subject to:} \quad s_n = \sum_{l \in \mathcal{L}: n = s(l)} x_l - \sum_{l \in \mathcal{L}: n = e(l)} x_l, \ \forall n \in \mathcal{N}, \tag{2}$$

$$r_c = \textstyle\sum_{l=1}^{L} R_{cl} x_l, \quad \forall c \in \mathcal{C}, \tag{3}$$

$$0 \leq x_l \leq x_l^{\max}, \quad \forall l \in \mathcal{L}, \tag{4}$$

$$s_n^{\min} \leq s_n \leq s_n^{\max}, \quad \forall n \in \mathcal{N}, \tag{5}$$

$$0 \leq r_c \leq r_c^{\max}, \quad \forall c \in \mathcal{C}. \tag{6}$$

Formally, the optimization variables are $\mathbf{x}$, $\mathbf{s}$, and $\mathbf{r}$. However, due to the existence of the equality constraints, one may adopt the view that the flows $\mathbf{x}$ are the only ones subject to optimization, and they affect the objective function through the "auxiliary" variables $\mathbf{s}, \mathbf{r}$.

The WMN must solve this problem in order to maximize the profit it makes out of transporting traffic minus the costs it pays to lease the channels and use the links (expression (1)), subject to the constraints that information is conserved (constraints (2)), the proper amounts of reservations are made (constraints (3)), and that flows, divergences, and reservation levels are between set bounds (constraints (4), (5), and (6).)

Note that the optimization problem is convex, as the utilities have been assumed concave, the channel and link costs convex, and all constraints are linear. Furthermore, the optimization function is separable, therefore this is a monotropic program, for which very efficient centralized solution methods exist [4].

Also note that we cast our problem as a single commodity optimization problem. Therefore, each packet inserted in the network does not have a specific destination, but may be received by any node whose divergence can be negative. This assumption makes sense for WMNs, where typically nodes are interested in communicating with any mesh router connected to the Internet, and the choice of mesh router is not important. However, the multicommodity case can also be formulated in a similar manner.

III. Distributed Flow Optimization using Duality

In this section we introduce a mechanism for solving Problem I that is distributed and can also be used to track the problem as the problem parameters change. To simplify the exposition, we now make the assumption that the utility function is strictly concave, and that both the link and channel cost functions are strictly convex [5]. The extension to the non-strictly convex case is straightforward, but is postponed for future work.

A. The dual problem

First, let us hypothetically assume that nodes are allowed to violate the data conservation constraint (2) provided they sell any excess traffic that gets accumulated at them at a **node price** $\lambda_n \in \mathbb{R}$, for node n, or buy any traffic they lack at that same price. This transaction is done with an external, hypothetical market. The resulting profit (or loss) is added to the objective. Let $\boldsymbol{\lambda} = (\lambda_n : n \in \mathcal{N})$ be the **node price vector**.

Secondly, let us assume that the network is allowed to violate constraint (3), and so reserve more or less channel bandwidth than the bandwidth needed. However, any excess, unused bandwidth must be sold off to the external market for a **channel price** $\mu_c \in \mathbb{R}$, for channel c, and any missing bandwidth must be bought at the same price. The resulting profit

(or loss) is also added to the objective. Let $\boldsymbol{\mu} = (\mu_c : c \in \mathcal{C})$ be the **channel price vector**.

We define the **Lagrangian** as the modified objective:

$$\begin{aligned} & L(\mathbf{x}, \mathbf{s}, \mathbf{r}; \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ & \triangleq \sum_{n=1}^{N} U_n(s_n) - \sum_{c=1}^{C} f_c(r_c) - \sum_{l=1}^{L} h_l(x_l) \\ & \quad + \sum_{n=1}^{N} \lambda_n \left[s_n - \sum_{l:s(l)=n} x_l + \sum_{l:e(l)=n} x_l \right] \\ & \quad + \sum_{c=1}^{C} \mu_c \left[r_c - \sum_{l=1}^{L} R_{cl} x_l \right] \\ & = \sum_{l=1}^{L} \left[\left(\lambda_{e(l)} - \lambda_{s(l)} - \sum_{c=1}^{C} R_{cl} \mu_c \right) x_l - h_l(x_l) \right] \\ & \quad + \sum_{n=1}^{N} \left[U_n(s_n) + \lambda_n s_n \right] + \sum_{c=1}^{C} \left[\mu_c r_c - f_c(r_c) \right]. \quad (7) \end{aligned}$$

Based on the above, the new optimization problem becomes

Problem 2: Relaxed U-C Problem

$$\begin{aligned} \text{maximize:} \quad & L(\mathbf{x}, \mathbf{s}, \mathbf{r}; \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{subject to:} \quad & 0 \le x_l \le x_l^{\max}, \quad l \in \mathcal{L}, \\ & s_n^{\min} \le s_n \le s_n^{\max}, \quad n \in \mathcal{N}, \\ & 0 \le r_c \le r_c^{\max}, \quad c \in \mathcal{C}. \end{aligned}$$

Observe, from expression (7), that the Lagrangian can be written as the sum of L terms each involving a single flow, N terms each involving a single divergence, and C terms each involving a single reservation level. Therefore, Problem 2 breaks into the following $L + N + C$ problems:

Problem 3a: Links ($l = 1, \ldots, L$)

$$\begin{aligned} \text{maximize:} \quad & \left(\lambda_{e(l)} - \lambda_{s(l)} - \textstyle\sum_{c=1}^{C} R_{cl} \mu_c \right) x_l - h_l(x_l), \\ \text{subject to:} \quad & 0 \le x_l \le x_l^{\max}. \end{aligned}$$

Problem 3b: Nodes ($n = 1, \ldots, N$)

maximize: $U_n(s_n) + \lambda_n s_n$, subject to: $s_n^{\min} \le s_n \le s_n^{\max}$.

Problem 3c: Channels ($c = 1, \ldots, C$)

maximize: $\mu_c r_c - f_c(r_c)$, subject to: $0 \le r_c \le r_c^{\max}$.

Observe that, due to the strict convexity of the costs and the strict concavity of the utility functions, each of these problems has a unique solution.

Let the **dual function** $q(\boldsymbol{\lambda}, \boldsymbol{\mu})$ be the maximum objective of Problem 2, i.e.,

$$q(\boldsymbol{\lambda}, \boldsymbol{\mu}) \triangleq \operatorname*{argmax}_{\mathbf{x}, \mathbf{s}, \mathbf{r}} L(\mathbf{x}, \mathbf{s}, \mathbf{r}; \boldsymbol{\lambda}, \boldsymbol{\mu}).$$

Consider also the following, **dual problem**:

Problem 4: Dual Problem

minimize: $q(\boldsymbol{\lambda}, \boldsymbol{\mu})$.

In this problem, the optimization is over $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ and there are no constraints. By standard duality theory [5], the optimum value of Problem 4 (which is a minimization) coincides with the optimum value of Problem 1 (which is a maximization); furthermore, the optimum values $\mathbf{x}^*, \mathbf{s}^*, \mathbf{r}^*$ can be found also by solving Problem 2, using for $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ the optimum values, $\boldsymbol{\lambda}^*$ and $\boldsymbol{\mu}^*$, of Problem 4. Therefore, we can solve Problem 1 by solving Problem 4, and then using its solution, $\boldsymbol{\lambda}^*$ and $\boldsymbol{\mu}^*$, to solve the (much simpler) Problem 2, through Problems 3a, 3b, 3c.

B. Distributed solution of time-invariant problem

In order to solve Problem 4 (and subsequently Problem 1), we note that, by standard duality theory and the strict convexity of the problem (Prop. 6.1.1, [5]), the dual function is continuously differentiable, and for each pair of price vectors $\boldsymbol{\lambda}, \boldsymbol{\mu}$, the gradient $\nabla q(\boldsymbol{\lambda}, \boldsymbol{\mu})$ of the objective function $q(\boldsymbol{\lambda}, \boldsymbol{\mu})$ can be found using the constraint violations of Problem 1. In particular, at the prices $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, the value of the component ∇q_{λ_n} of the gradient corresponding to the price λ_n is

$$\nabla q_{\lambda_n}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = s_n - \sum_{l \in \mathcal{L}: n = s(l)} x_l + \sum_{l \in \mathcal{L}: n = e(l)} x_l, \quad \forall n \in \mathcal{N}, \tag{8}$$

and the value of the component ∇q_{μ_c} of the gradient corresponding to the price μ_c is

$$\nabla q_{\mu_c}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = r_c - \sum_{l=1}^{L} R_{cl} x_l, \quad \forall c \in \mathcal{C}, \tag{9}$$

where the vectors $\mathbf{x}, \mathbf{s}, \mathbf{r}$ are the solutions of Problems 3a, 3b, 3c, for the prices $\boldsymbol{\lambda}, \boldsymbol{\mu}$.

These formulae can be used for the distributed implementation of a gradient descent algorithm solving Problem 4. The algorithm will be executed through the actions of a **link agent** l (LAl) for each link l, a **node agent** n (NAn) for each node n, and a **channel agent** c (CAc) for each channel c.

Distributed U-C Algorithm

INPUT: Each agent naturally possesses some of the problem parameters it will need later. In particular:

1) LAl knows $h_l(\cdot)$, $x_l^{\max}$, and the coefficients R_{cl} for all channels c it interferes with.
2) NAn knows $U_n(\cdot)$, $s_n^{\min}$, and $s_n^{\max}$.
3) CAc knows $f_c(\cdot)$ and $r_c^{\max}$.

STEP 1 (INITIALIZATION) Let the iteration $i = 0$. NAn selects an arbitrary initial λ_n^0 and relays it to the agents of its incident links. Also, CAc selects an arbitrary initial μ_c^0 and relays it to the links it is being interfered from.

STEP 2 Set $i = i + 1$.

1) LAl solves the l-th Problem 3a and sends the flow x_l^i found to the node agents $s(l)$ and $e(l)$ and the channel agents of all channels it interferes with.
2) NAn solves the n-th Problem 3b and finds s_n^i.

3) CAc solves the c-th Problem 3c and finds r_c^i.

STEP 3

1) NAn finds a new value for the price λ_n^i, using
$$\lambda_n^i = \lambda_n^{i-1} - a\nabla q_{\lambda_n}^i.$$
a is a globally agreed step size and $\nabla q_{\lambda_n}^i$ is found using (8). NAn then relays λ_n^i to the agents of its incident links.
2) CAc find a new price μ_c^i, using
$$\mu_c^i = \mu_c^{i-1} - a\nabla q_{\mu_c}^i.$$
$\nabla q_{\mu_c}^i$ is found using (9). CAc then relays μ_c^i to the links it is being interfered from.

STEP 4 If $i < I$, where I is a globally known maximum, then GOTO STEP 2.

STEP 5 Produce as output the flows $\mathbf{x}$, $\mathbf{s}$ and $\mathbf{r}$ found by solving Problems 3a, 3b, 3c, with the last prices $\boldsymbol{\lambda}^I$ and $\boldsymbol{\mu}^I$.

Observe that communication is constrained between agents that are physically near. As this mechanism implements a form of gradient descent, it is guaranteed to converge [5], in the sense that as I increases, the last set of prices λ_n^I, μ_c^I converge to their optimal values λ_n^*, μ_c^*, and the flows produced at the output also converge to their optimal values.

C. The case of variable problem parameters

The algorithm of Section III-B assumes that the parameters of the problem (i.e., the utility and cost functions and the various bounds) remain fixed. However, it can easily be adapted to changing conditions. The way to do this is to make sure that, during STEP 2, the most updated versions of Problems 3a, 3b, 3c are solved. For example, if the utility function of a node changes, the agent of that node will have to adjust its problem formulation, and start solving a different problem.

Formally, let us assume that the variables appearing at the INPUT are changing with time, and so with the iteration i. With respect to the Distributed U-C Algorithm, the following changes are necessary:

1) At each STEP 2, the latest version of the input is used.
2) We remove the termination condition, so that the STEP 2-STEP 4 loop is executed continuously.
3) At the end of STEP 2, the agents use the prices currently available to them to construct, in a distributed manner, a feasible, suboptimal solution $\mathbf{x}'$, $\mathbf{s}'$, $\mathbf{r}'$.

We refer to the resulting algorithm as the **Variable Input Distributed U-C Algorithm**.

The distributed construction of a feasible solution at the end of STEP 2, is necessary because solving the Problems 3a, 3b, and 3c with any prices other than the optimal ones will in general provide us with a set of infeasible vectors $\mathbf{x}, \mathbf{s}, \mathbf{r}$. Under non-trivial scenarios, we expect that the algorithm will never converge to the optimum values, and therefore the vectors it will be providing, while hopefully close the optimum, will be infeasible. Due to space limitations, we present a distributed algorithm for constructing such a feasible solution in [6].

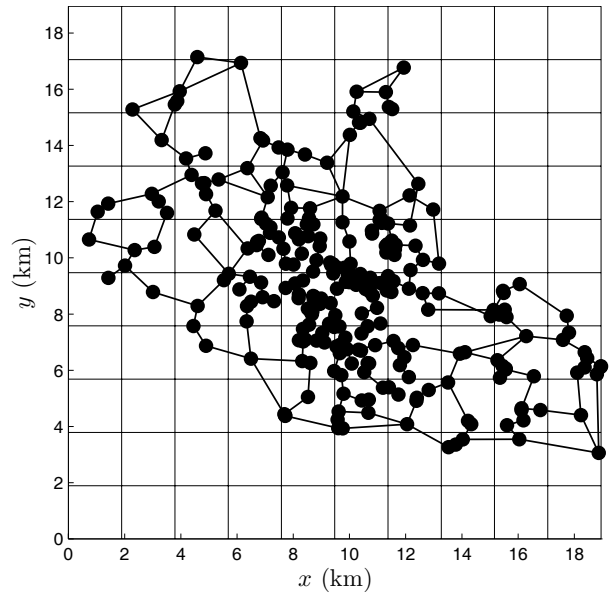

Fig. 1. The WMN example of Section IV.

IV. Simulations

We offer a preliminary evaluation of the distributed mechanism in a synthetic scenario based on the Funkfeuer WMN deployed in Vienna, real-life mobility traces of taxis collected at the same city, and simulated input regarding the CN channels and the WMN connectivity, among others. The simulation setup does not comply to reality fully, and so should not be seen as a quantitative evaluation of the potential real-life synergy between a WMN and a CN, but as a proof of concept.

Our WMN contains 265 nodes placed in the actual locations of Funkfeuer [3] routers in the Vienna. There are 592 links, chosen so that the resulting connectivity closely resembles that of actual WMNs, with a combination of short range and long range links. The region containing the routers is covered with 100 CN cells arranged in a square grid. Frequency reuse with a reuse factor of 4 is assumed. The WMN supports the download of information by a number of vehicular users, as we describe shortly.

We specify the reservation matrix as follows:

1) If link l connects two nodes s, e that exist on the same cell c, then we set $R_{cl} = 1$.
2) Let link l connect a transmitter node s and a receiver node e lying on adjacent cells that share an *edge*. Let c_1 be the cell where the transmitter is located, and let c_2 the cell that shares the same frequency set with c_1 and is closest to the receiver node e. We set $R_{c_1 l} = R_{c_2 l} = 1$.
3) Let link l connect a transmitter node s and a receiver node e lying on adjacent cells sharing only a common *vertex*. Let c_1 be the cell where the transmitter is located, and let c_2, c_3, c_4 the three cells that share the same frequency set with c_1 and are closest to the receiver e. We set $R_{c_1 l} = R_{c_2 l} = R_{c_3 l} = R_{c_4 l} = 1$.
4) All other entries of the reservation matrix are set to zero.

Regarding the channel costs, we set $f_c(r_c) = r_c^2$, and also $0 \leq r_c \leq r^{\max} = 50$. Therefore, the prices of the CN bandwidth becomes steeper as the WMN reserves more of it, and the CN is forced to operate closer to its capacity. Regarding the link costs, we set $h_l(x_l) = x_l^2$, and also $0 \leq x_l \leq x^{\max} = 5$. Our motivation for using this link cost is that it promotes load balancing, as links with large traffic volumes are heavily penalized.

We are simulating a downloading scenario as follows: Among the nodes, one, centrally located, is chosen as a gateway. Let this be node 1. We set $s_1^{\min} = 0$, $s_1^{\max} = 26500$, so that the gateway can only insert traffic. All other nodes are required to remove traffic, and in particular we set $s_n^{\min} = -100$ and $s_n^{\max} = 0$ for all $n \neq 1$.

The utility of the gateway is chosen to be $U_1(s_1) = 0$. To establish the utility function of the other nodes, we use real life mobility traces corresponding to approximately 1300 taxis equipped with GPS receivers operating in the greater Vienna region as follows. With each mobility trace we associate a vehicle interested in downloading data from the gateway. Let $k(n)$ be the number of vehicles that are within 500 m of node $n \neq 1$ and closer to node n than to any other. Due to mobility, $k(n)$ is a function of time. For our simulations, we used samples of $k(n)$ separated by one minute intervals, over a five hour period.

Let $u(\cdot)$ be the utility function *of a single vehicle* downloading data. We set it to be $u(x) = 1000 \log(1 - x)$, $x \leq 0$. If there are $k(n)$ vehicles associated with the node n, then the utility function *for that node* is

$$U_n(s_n) = \begin{cases} k(n)u(s_n/k(n)), & s_n \leq 0, \\ -\infty, & s_n > 0. \end{cases}$$

The rationale is simple: as $u(\cdot)$ is concave, it is best to divide the divergence in equal parts to all $k(n)$ vehicle. Each vehicle derives a utility equal to $u(s_n/k(n))$, and all $k(n)$ of them a utility $k(n)u(s_n/k(n))$.

As the vehicles are moving, $k(n)$ is changing with time, and with it the utility function of node n and consequently the optimal traffic. Indeed, in the first plot of Fig. 2 we plot, with a continuous line, the value of the optimum of Problem 1, as obtained using MATLAB, as it changes with time. The x-axis is in minutes, and every single minute the numbers of taxis $k(n)$ change. In the same figure, we have plotted, with dashed lines, the dual function and the corresponding feasible value obtained if the prices are updated 20 times every minute. (The dual values are above the optimum, and the feasible values below it.) Finally, we have plotted, with dotted lines, the dual function and the corresponding feasible value obtained if the prices are updated 200 times every minute. The behavior of the algorithm can be seen more clearly in the second plot of Fig. 2, where we have focused in a time window where the algorithm has converged from its initial point of operation to near-optimal operation points.

It is apparent from these figures that our method leads to the calculation of such flows that the corresponding total benefit is constantly close to the optimal one, except for the initial "bootstrapping" period. As expected, this tracking is more efficient if more iterations are possible before the problem changes.

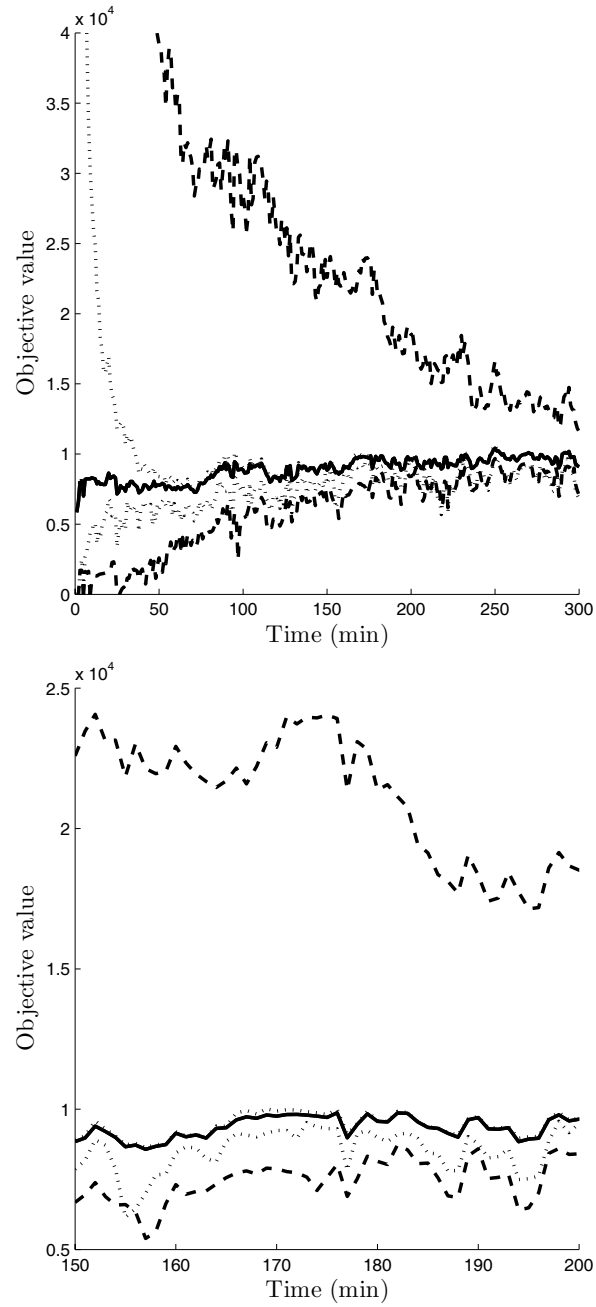

Fig. 2. Evolution of the distributed mechanism in the time intervals [0 min, 300 min] and [150 min, 200 min]. The optimal objective is denoted by a continuous line. The dual value and the corresponding feasible value for 20 price updates per minute (200 updates per minute) are plotted with dashed (dotted) lines, over and below the optimal objective respectively.

V. Related Work

Ever since its introduction [7], cognitive radio has been identified as a key communication paradigm capable of pro-

viding substantial relief to the problem of bandwidth shortage. A review of Cognitive Network research appears in [1].

One of the first works to study Cognitive Wireless Mesh Networks is the work in [8]. There, a new approach to spectrum sensing is devised, an analytical framework is proposed for allowing mesh routers to estimate the activity in a channel, and a channel assignment problem is formulated. In contrast to our work, however, the primary channel is used to support the communication between mesh clients and mesh routers, and not the communication between the routers.

Closer to our work is the work in [9]. There, the authors present a formulation for performing fair bandwidth allocation in a cognitive WMN. Their formulation performs simultaneously routing, scheduling, and spectrum allocation, and so is similar to our own. However, the two approaches differ in the following important issues: First, in [9] centralized algorithms are proposed. Secondly, the optimization problems formulated are linear and achieve either max-min or lexicographic max-min fairness. Finally, the work in [9] does not scale well, as it requires the explicit calculation of all modes of operation in the network.

In [10] the authors develop a multi-commodity formulation for minimizing the network-wide use of the primary radio bandwidth. A Mixed Integer Nonlinear Program is formulated. Due to its complexity, the authors propose a method for establishing tight lower and upper bounds for the objective. In follow-up work [11], a distributed algorithm for approximately solving a similar, also MINLP, optimization problem is proposed.

In [12] the authors consider a stochastic setting in which the availability of bandwidth, caused by the absence of primary users, is random, and not known beforehand to the mesh nodes. The authors introduce an optimization problem that can be solved by a distributed algorithm which requires no prior knowledge of the probabilistic behavior of the primary users. There are a number of differences to our formulation, the most critical being that transmissions of secondary users over the same primary bandwidths do not interfere with each other.

Note that, in contrast to the majority of other cognitive radio formulations, where the primary users are oblivious to the existence of secondary users (the *commons* model) we assume that the primary user, i.e., the CN, is actively cooperating with the secondary user, i.e., the WMN, by leasing its spare bandwidth (the *spectrum leasing* model). Other works focusing on this approach are [13], [14], [15].

Finally, we mention that a formulation related, but distinct, to the one proposed here was succinctly delineated, by the authors of this work and others, in [16]. There, no distributed algorithm was presented, and the emphasis was on the study of multiple WMNs competing for the same resources.

VI. Conclusions

We present a framework for enabling a potentially powerful synergy between WMNs and CNs. We present an optimization problem that a WMN can solve to optimize its traffic given its own utilities and costs and the cost of leasing cellular bandwidth. We present a distributed algorithm that solves this problem even while the problem parameters change. A preliminary numerical study verifies that the mechanism manages to provide traffic flows close to the optimal ones.

Future work includes the development of a detailed packet level simulator that can provide quantitative evaluation of a real-life synergy between a WMN and a CN; the extension of the theory in the multicommodity case; the development of faster iterative algorithms such as Newton and quasi-Newton methods; the extension of the formulation in the plain convex and non-convex cases; and the game-theoretic investigation of multiple WMN, multiple CN scenarios.

Acknowledgment

Research for this work has been partly funded by MOMO (Multi-Overlays for Multi-hOming) project, a SJRP (Specific Joint Research Project) within the EURO-NF Network of Excellence (NoE).

References

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, 50(13), pp. 2127–2159, Sep. 2006.

[2] D. Willkomm, s. Machiraju, J. Bolot, and A. Wolisz, "Primary user behavior in cellular networks and implications for dynamic spectrum access," *IEEE Communications Magazine*, 47(3), pp. 88–95, Mar. 2009.

[3] [Online]. Available: www.funkfeuer.at

[4] D. P. Bertekas, *Network Optimization: Continuous and Discrete Models*, 1st ed. Belmont, MA: Athena Scientific, 1998.

[5] ——, *Nonlinear Programming*, 2nd ed. Belmont, MA: Athena Scientific, 1999.

[6] S. Toumpis, I. Tselekounis, G. Stamoulis, H. Meyer, A. Hess, and K. A. Hummel, "Cognitive WMN: A distributed mechanism for leasing cellular network bandwidth," extended technical report, available online at http://pages.cs.aueb.gr/∼toumpis/hotmeshnet_extended.pdf.

[7] J. Mitola III and G. Q. Maguire, Jr., "Cognitive radio: Making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[8] K. R. Chowdhury and I. F. Akyildiz, "Cognitive wireless mesh networks with dynamic spectrum access," *IEEE Journal on Select. Areas in Communications*, vol. 26, no. 1, pp. 168–181, Jan. 2008.

[9] J. Tang, R. Hincapié, G. Xue, W. Zhang, and R. Bustamante, "Fair bandwidth allocation in wireless mesh networks with cognitive radios," *IEEE Trans. on Vehic. Technol.*, vol. 59, no. 3, pp. 1487–1496, Mar. 2010.

[10] Y. T. Hou, Y. Shi, and H. D. Sherali, "Spectrum sharing for multi-hop networking with cognitive radios," *IEEE Journal on Select. Areas Commun.*, vol. 26, no. 1, pp. 146–155, Jan. 2008.

[11] Y. Shi, Y. T. Hou, H. Zhou, and S. F. Midkiff, "Distributed cross-layer optimization for cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 8, pp. 4058–4069, Oct. 2010.

[12] Y. Song, C. Zhang, and Y. Fang.

[13] L. Duan, J. Huang, and B. Shou, "Cognitive mobile virtual network operator: Investment and pricing with supply uncertainty," in *Proc. IEEE INFOCOM*, San Diego, CA, Mar. 2010.

[14] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperative secondary ad hoc networks," *IEEE Journal on Select. Areas Commun.*, 26(1), pp. 203–213, Jan. 2008.

[15] S. K. Jayaweera and T. Li, "Dynamic spectrum leasing in cognitive radio networks via primary-secondary user power control games," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 6, pp. 3300–3310, June 2009.

[16] S. Sargento et al., "User-driven multi-access communications through virtualization," in *Wireless Multi-Access Environments and Quality of Service Provisioning: Solutions and Application*, G.-M. Muntean and R. Trestian, Eds. IGI Global, 2011, to appear, available online at http://pages.cs.aueb.gr/∼toumpis/momo_bookchapter.pdf